Solving Circuits Worksheets

Teachers Notes

The following series of worksheets will help the students work through the solutions of solving electric circuits. The difference with using the software is the student get feedback immediately if they didn't get it right. They can then quickly move to the required redraw and measure the values to see if they got them right.

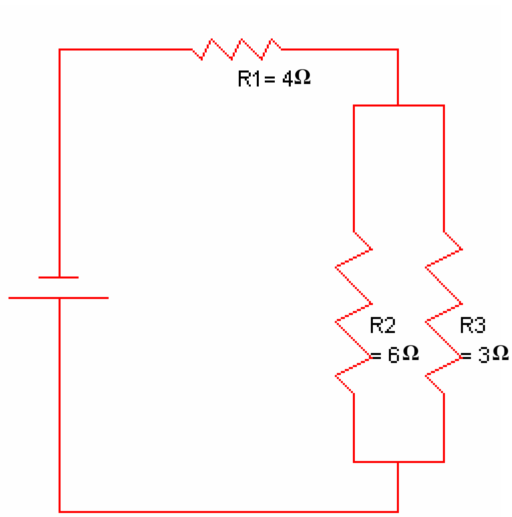
Solving circuits entails 4 rules.

1. Look for a part of the circuit you can simplify and reduce that part only.
2. Do not get "Lazy" in the redraws. Do not combine 2 simplifications into one. You won't be able to do step 4 if you do.
3. If you have 2 of the values use V = I·R to find the third
4. Carry the information that is the same back through the simplified circuits.

I find these rules give the kids a good starting point that enables them to solve some very difficult circuits.

Example:

Solve the circuit below if the total voltage is 12 V.



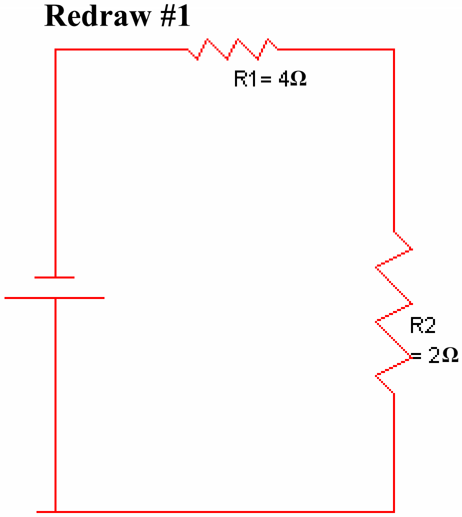
Rule #1→ One of the most common mistakes is that the students think R_1 and R_3 are in series. (Remind them that they are only in series if the circuit doesn't split up.) We realize that R_2 and R_3 are in parallel, so we use the formula $\frac{1}{R_T} = \frac{1}{R_2} + \frac{1}{R_3}$ to solve for R_T.

Rule #2→Redraw the circuit with this simplification

Rule #3→Doesn't apply yet

Rule #4→Doesn't apply yet

Redraw #1



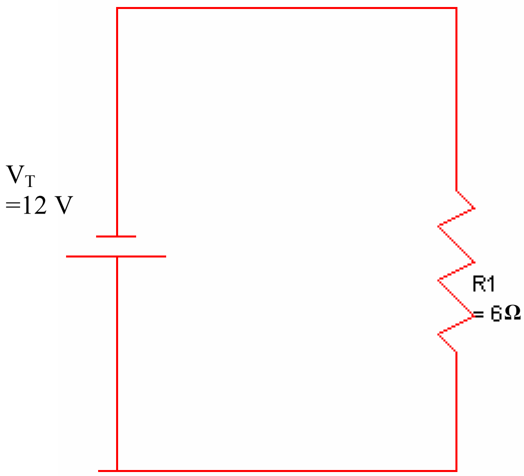
Rule #1→ R_1 and R_2 are in series. So we use $R_T = R_1 + R_2$ to solve for R_T.

Rule #2→Redraw the circuit with this simplification

Rule #3→Doesn't apply yet

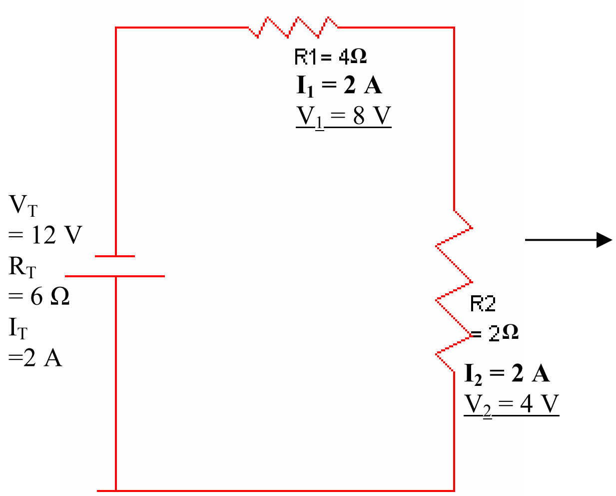
Rule #4→Doesn't apply yet

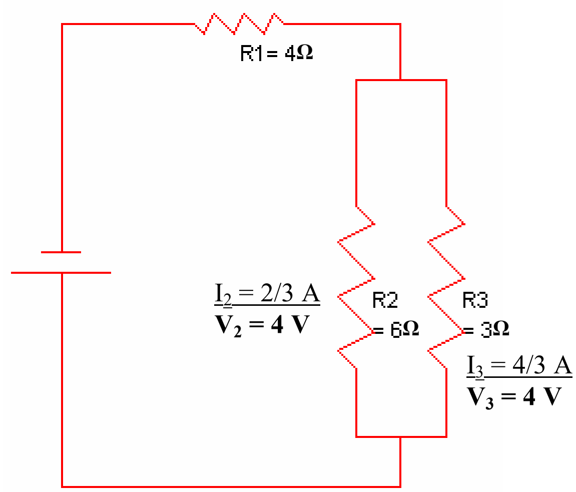
Redraw #2



We have now completed the redraws so rule #1 and #2 are not needed any more.
We now know that R_T for the circuit is 6Ω and since we know V_T = 12 V we can use rule #3. Using $V = I \cdot R$,
$12 = 6\ I_T$ therefore $I_T = 2$ A

Now is the time to start carrying the info back up to the other redraws. The bolded values are the ones that were transferred through. The underlined value was calculated.





Redraw #1

Original

This is a simple series circuit so we know that current is the same so the current through R_1 and R_2 is 2 A. This allows us to now use rule #3.
$R_1 = 4\ \Omega$, $I_1 = 2$ A $\therefore V_1 = I_1 \cdot R_1 = 2 \cdot 4 = 8$ V
$R_2 = 4\ \Omega$, $I_2 = 2$ A $\therefore V_2 = I_2 \cdot R_2 = 2 \cdot 2 = 4$ V

We can now go back to the original parallel part of the circuit and that in a parallel circuit the voltage is the same, we put the voltage in for those resistors. We can now solve for the last unknown currents.

Note: An example of where rule **Number 2** comes into play is as follows. The students will solve for the parallel and then know that the total is 6 ohms and not redraw the first redraw. Now they are “stuck” when working backwards. The stronger students will be able to do it but strongly recommend them to NOT skip the redraw. As the circuits can, and do, get more complicated and skipping a redraw can prevent them from finishing the question.

Circuit Theory Review

Kirchoff's Laws
These laws are used to help solve the analysis of circuits.

Kirchoff's laws can be summarized as follows:

- The current entering at a point must equal the current leaving the point. (This means that electrons are never "lost" in the circuit.)
- If you trace a path through the circuit, the energy (Voltage) gained by the batteries must equal the energy (Voltage) lost by the loads. (Conservation of Energy)

Voltage is an equivalent to Energy because the voltage can be determined by $V=\Delta E/Q$. The formula says that Voltage is a measure of the change in energy per unit of charge. So it is fair to say that for each charge passing through the circuit the energy gained and lost by that charge must be the same. And the energy is the voltage.

As well, we will use the electron flow convention for current, which refers to the direction that electrons travel in. This is in the opposite direction that conventional flow says but the logic is the same.

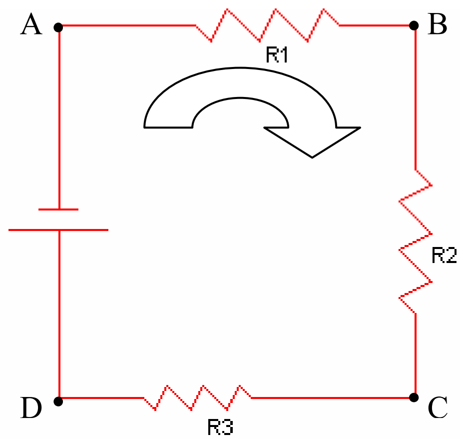
There are two basic circuits**. Series and Parallel.**

We will use Kirchoff's Laws to help determine the relationship of the total Voltage, Current and Resistance for the circuit.

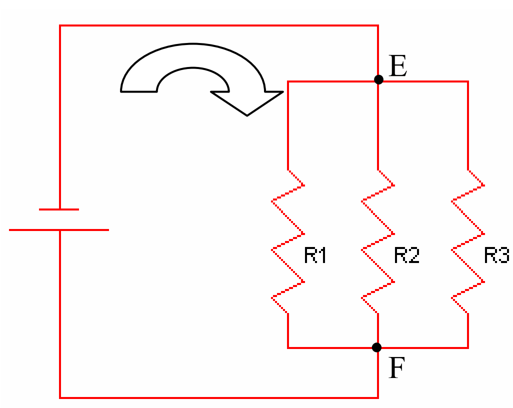
The notation we will use is quite straightforward. The current going through $\mathbf{R_1}$ is called $\mathbf{I_1}$ and the voltage across $\mathbf{R_1}$ is called $\mathbf{V_1}$. Likes wise for R_2 and R_3. For the total circuit we use Variables $\mathbf{V_T}$, $\mathbf{I_T}$ and $\mathbf{R_T}$.

To develop the relationships of the 3 values in the circuits, we will compare the two types side by side so you can see the similarities in the development.

Series Circuit (in a "straight line")



Parallel Circuit (A "Ladder" shape)



To analyze the circuit we will follow the circuit in the direction of the large arrows.

Series

We gain energy by going through the battery and we pass through all three resistors.

We can then state that
$V_T = V_1 + V_2 + V_3$.

If we now look at the currents involved, we notice that the currents at each dot on the circuit must be the same. Current entering at point **A** is the same as the current leaving. This is also true for points **B**, **C** and **D**. No electrons are "lost" through the circuit, so the currents through each resistor must be the same. We can then state that $I_T = I_1 = I_2 = I_3$.

Now using $V_T = V_1 + V_2 + V_3$ and then substituting in $V = I \cdot R$, we get
$I_T \cdot R_T = I_1 \cdot R_1 + I_2 \cdot R_2 + I_3 \cdot R_3$.

Since all the currents through each resistor matches the total current.

The equation becomes
$I_T \cdot R_T = I_T \cdot R_1 + I_T \cdot R_2 + I_T \cdot R_3$.

Now factoring
$I_T \cdot R_T = I_T \cdot (R_1 + R_2 + R_3)$

And then dividing through by I_T we get
$R_T = R_1 + R_2 + R_3$.

This equation makes intuitive sense. The more resistors added in series the larger the total resistance.

Parallel

We gain energy by going through the battery but we notice that whatever path we take, we only go through one Resistor.
We can then state that
$V_T = V_1 = V_2 = V_3$ since the Voltage gained by the battery must be lost by the voltage of the resistor that we pass through.
If we now look at the currents in the circuit we notice that the current entering at point **E** must be split into 3. Since the 3 paths lead out of the point and through the three resistors, we can say
$I_T = I_1 + I_2 + I_3$.
We will now substitute in for the I's by using a rearranged equation of $V = I \cdot R$.
So $I = V/R$

$$\frac{V_T}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}.$$

Since we know that the voltages across each resistor is the same

$$\frac{V_T}{R_T} = \frac{V_T}{R_1} + \frac{V_T}{R_2} + \frac{V_T}{R_3}$$

And once again factoring and then dividing by the common term,

$$\frac{V_T}{R_T} = V_T\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

This equation isn't quite as intuitive. The more resistors you have in parallel the lower the total resistance.

This means that in a series circuit

- the total current is the same throughout
- the sum of voltage is equal to the total voltage
- the sum of resistance is equal to the total Resistance

This means that in a parallel circuit

- the sum of currents equals the total Current
- the total voltage is the same throughout
- The sum of the reciprocals of the resistance is equal to the reciprocal of the total resistance

Summary

	Series	Parallel
Currents	$I_T = I_1 = I_2 = I_3$	$I_T = I_1 + I_2 + I_3$
Voltage	$V_T = V_1 + V_2 + V_3$	$V_T = V_1 = V_2 = V_3$
Resistance	$R_T = R_1 + R_2 + R_3$	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

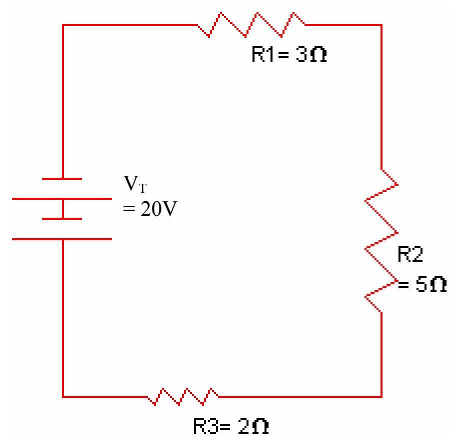
Using the Formulas

Now we need to get familiar with how to use the formulas.
Here is some simple sample questions with solutions. More advanced questions are involved in other worksheets.

For the following circuits fill in the missing information:

1)

	Total	1	2	3
Voltage	**20 V**			
Current				
Resistance		**3 Ω**	**5 Ω**	**2 Ω**

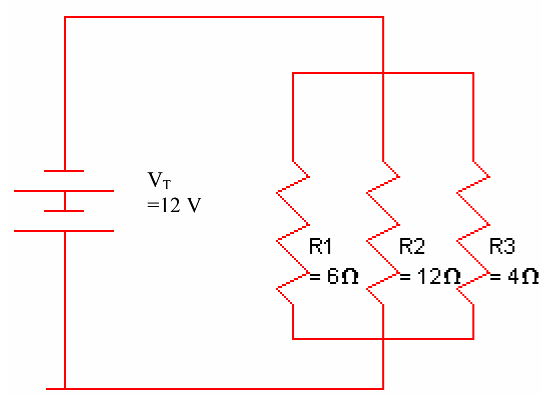


Order of solution

Solve for R_T: Since it is in series, $R_T = R_1 + R_2 + R_3$ therefore $R_T = 10\ \Omega$.
We can now solve for I_T. Using $V = I \cdot R$ we get $I_T = 2$ A.
Since this is a series circuit, the currents in each resistor are the same. So $I_1 = 2$ A, $I_2 = 2$ A and $I_3 = 2$ A.
This now allows us to solve for Voltages.
$V_1 = 6$ V, $V_2 = 10$ V and $V_3 = 4$ V.
The nice thing about circuit work is that there is a self-check. We know that through any path, the sum of the voltages must equal the voltage across the battery. A quick total shows that it does equal total voltage.

2)

	Total	1	2	3
Voltage	**12V**			
Current				
Resistance		**6 Ω**	**12 Ω**	**4 Ω**



Order of solution

Solve for R_T: Since it is in series, $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

therefore $R_T = 2\ \Omega$.
We can now solve for I_T. Using $V = I \cdot R$ we get $I_T = 6$ A.
Since this is a parallel circuit, the Voltages in each resistor are the same. So $V_1 = 12$ V, $V_2 = 12$ V and $V_3 = 12$ V.
This now allows us to solve for Currents.
$I_1 = 2$ A, $I_2 = 1$ A and $I_3 = 3$ A.
Now for the self-check. For a simple parallel circuit we know that the current leaving the battery must equal the sum of the currents through each resistor. A quick total shows that it does equal total current.

To gain familiarity with the program, go check the measurements by using the multi-meter available in the program. The circuits are already stored in the program. Go to **Circuits→work sheets→series→circuit #1** and **Circuits→work sheets→parallel→circuit #1**.

Using the Multi-meter

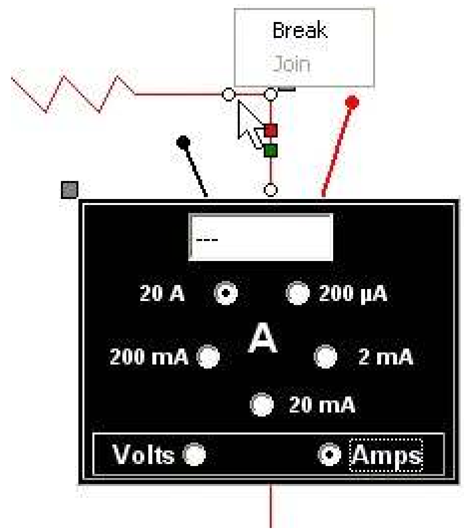
Open one of the circuits and then turn the meter on. Select **Meter→On**. You can move the meter by dragging the grey square at the top right corner of the meter.

Voltmeter

To use the voltmeter simply drag the red and black leads on top of the white circles, located throughout the circuit, on opposite sides of the resistor you want to measure. Change the scale on the meter to show how accuracy changes. The dial on the meter tells the largest value it can read. If the meter reads "**--0--**" the scale is too low and it needs to be increased.
The voltmeter measures the change in voltage from one side of the resistor to the other so a negative value means that energy is lost. A positive value simply means the meter is hooked up backwards but doesn't affect the total. A battery typically should be a positive value since it adds energy and a resistor/bulb should be negative since it loses energy.

Ammeter

The ammeter requires more care in its use. You need to put the meter in the circuit. (If you use the meter like a voltmeter you will "blow" a fuse and the screen will go black. Go to **Meter→Change fuse** to change the fuse.) So you need to break the circuit where you want to put the meter. To do this, simply click on the white circle where you want to insert the meter and select break. (After taking the measurements, click the open part of the circle and choose join.) Now insert the meter on both side of the break.
Once again try different scales on the meter and see how the accuracy changes.



Using the premade Circuits

You can easily modify each of the premade circuits. However, if you do use the circuits found under the worksheet option and you make changes and then move to the next circuit and then back again the modification to the circuit is not saved. To save the changes simply go to **Save→New file**. If you want to add the file to already presaved file you can simply go to **Save→Add to file**.

Using $\frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$ Formula

It is great practice for the students to use fraction skills to solve the formula. i.e. find common denominator, as well as have them quickly use the $\boxed{x^{-1}}$ or $\boxed{1/x}$ buttons on their calculator to quickly solve it.